

Quantum Computation and Bell-state Measurement with Double-Dot Molecules

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We propose a quantum computation architecture of double-dot molecules, where the qubit is encoded in the molecule two-electron spin states. By arranging the two dots inside each molecule perpendicular to the qubit scaling line, the interactions between neighboring qubits are largely simplified and the scaling to multi-qubit system becomes straightforward. As an Ising-model effective interaction can be expediently switched on and off between any two neighboring molecules by adjusting the potential offset between the two dots, universal two-qubit gates can be implemented without requiring time-dependent control of the tunnel coupling between the dots. A Bell-state measurement scheme for qubit encoded in double-dot singlet and triplet states is also proposed for quantum molecules arranged in this way.

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I. INTRODUCTION

In 1998, D. Loss and D. P. DiVincenzo proposed a quantum computation (QC) protocol based on electron spin trapped in semiconductor quantum dot[1]. Compared with other systems such as optics, atoms and nuclear magnetic resonance (NMR), this solid system is argued to be more scalable and can be compatible to the present semiconductor technology[2]. Recently, two-electron spin states in double quantum dots have attracted many interests[3, 4, 5]. The Initialization, manipulation and detection of these two-electron spin states have been theoretically analyzed and experimentally demonstrated[6, 7, 8]. Then there is the idea to encode qubit on the singlet state $S = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ and the triplet state $T = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ of double coupled quantum dots[9]. A fault-tolerant architecture for QC is also proposed for these two-spin qubits[9]. It is argued that this encoding can protect qubits from low-frequency noise, and suppresses the dominant source of decoherence from hyperfine interactions[6, 10, 11, 12, 13, 14]. In the papers[15, 16], the quantum molecules are arranged in line and the two dots inside each molecule are also arrayed in the same line as shown in Fig.1(a). The qubit is encoded in the singlet and triplet states of the double dots inside each molecule and the neighboring qubits are coupled by the direct Coulomb repulsion of the electrons between different molecules. Two-qubit controlled-not gate are analyzed in details when four quantum dots (two molecules) are ranged in line.

Different to the previous one-dimensional alinement of all quantum dots[9, 15, 16], we here propose an architecture to arrange: the two dots inside each molecule perpendicular to the qubit scaling line as Fig.1(b). As the qubit is encoded in the two-electron spin states of each molecule, an Ising-model effective interaction can

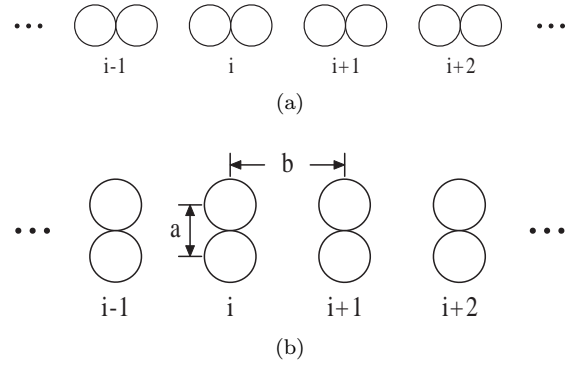


FIG. 1: Schematic diagram of double-dot qubit system. All quantum dots have same size, and their radius is denoted by r . (a) The structure that all quantum dots are arranged in line. (b) The structure that the two dots inside each molecule perpendicular to the qubit scaling line. The distance between two dots of each double dot is a , and the distance between two double dots is b .

be switched on and off between any two neighboring molecules without affecting other neighboring qubits in this architecture. Universal two-qubit gates can be implemented without requiring time-dependent control of the tunnel coupling between the dots. A Bell-state measurement for qubit encoded in double-dot singlet and triplet states is also analyzed for quantum molecules arranged in this way.

In the section II, we analyze the qubit initialization and single qubit operations. The realization of two-qubit gate operations is detailedly investigated in section III. Section IV includes the single qubit readout and Bell state measurement. In the last section, we give some discussions and present our conclusion.

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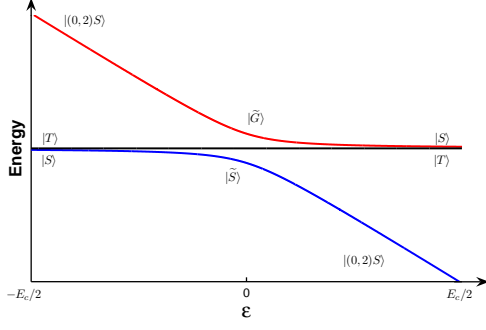


FIG. 2: Energy level structure of the double dot system. The range of the bias voltage ε which we interested in is between $-E_c/2$ and $E_c/2$.

II. INITIALIZATION AND SINGLE QUBIT OPERATIONS

Consider two double-dot molecules as shown in Fig.1(b). The two dots inside each molecule perpendicular to the qubit scaling line. Inside each quantum molecule, there are generally three energy favorable states $|(1,1)S\rangle$, $|(1,1)T\rangle$ and $|(0,2)S\rangle$ included for coupled double-dot due to Coulomb blockade and Pauli blockade[6, 7, 8, 17]. The notation (n_u, n_l) indicates n_u electrons on the “upper” dot of each qubit and n_l electrons on the “lower” dot. Define a parameter ε to represent potential offset between two dots inside each molecule, which can be changed by external electrical field or bias voltage on the gates defining quantum dots[18]. The range of ε can be changed between $-E_c/2$ and $E_c/2$, and for this case, there are only two charge states of each qubit: (0,2) and (1,1). Here E_c indicates the charging energy of each dot. Due to Pauli blockade, the double-dot state can shift to charge state (0,2) if the initial state is $|(1,1)S\rangle$ (we use $|S\rangle$ to denote $|(1,1)S\rangle$ in the following text), but remain in charge state (1,1) if the initial state is $|T\rangle$. The energy level structure of each molecule is shown in Fig.2. Due to the tunneling between the two dots, the charge states (0,2) and (1,1) hybridize. According to reference [8], we can define two superposition states

$$\begin{aligned} |\tilde{S}\rangle &= \cos\theta |S\rangle + \sin\theta |(0,2)S\rangle, \\ |\tilde{G}\rangle &= -\sin\theta |S\rangle + \cos\theta |(0,2)S\rangle. \end{aligned} \quad (1)$$

where

$$\theta = \arctan\left(\frac{2T_c}{\varepsilon - \sqrt{4|T_c|^2 + \varepsilon^2}}\right). \quad (2)$$

Here T_c indicates the tunnel coupling. By adiabatically sweeping ε from $-E_c/2$ to $E_c/2$, the double-dot states $|\tilde{S}\rangle$ and $|\tilde{G}\rangle$ evolve according to Eq. 1. The rapid

adiabatic passage means that the transformation of ε is fast relative to the nuclear mixing time $\sim \hbar/(g^*\mu_B B_{nuc})$ but slow with respect to the tunnel coupling T_c [6, 18]. Here g^* is the effective g -factor of the electron and μ_B is the Bohr magneton. When $\varepsilon = -E_c/2 \ll -|T_c|$, the adiabatic angle $\theta \rightarrow 0$, and the eigenstate $|\tilde{S}\rangle \rightarrow |S\rangle$, $|\tilde{G}\rangle \rightarrow |(0,2)S\rangle$. When $\varepsilon = E_c/2$, we get $\theta \rightarrow \pi/2$ and the eigenstates $|\tilde{S}\rangle \rightarrow |(0,2)S\rangle$, and $|\tilde{G}\rangle \rightarrow |S\rangle$. Thus by adiabatically sweeping ε from $-E_c/2$ to $E_c/2$, the double dots initially in the state $|S\rangle$ can change from the charge state (1,1) to (0,2). For simplicity, we use $|S'\rangle$ to represent the singlet state $|(0,2)S\rangle$ in the following text.

We can initialize the system to the state $|S\rangle$ by loading two electrons from a nearby Fermi sea into the ground state of a single quantum dot $|S'\rangle$ and then sweeping the bias voltage ε from $E_c/2$ to $-E_c/2$ in the rapid adiabatic passage to change the charge state from (0,2) to (1,1)[6, 8, 9]. In the following, we can see that when the neighboring qubits are both in the charge state (0,2), the interaction between them will be switched on. Thus, the initialization can only be simultaneously made on non-neighboring qubits. For a one-dimensional qubit array, we need at least two steps to initialize all qubit to the (1,1) charge state $|S\rangle$. For a two-dimensional array, four steps are needed. After initialization, the bias voltage ε of all qubits are kept in the value of $-E_c/2$ and all qubit charge states are in (1,1).

According to the Euler angle method, if rotations by arbitrary angles about two orthogonal axes are available, arbitrary single-qubit rotations can be constructed. For the present double-dot molecule, it has also been analyzed and experimentally shown that arbitrary single-qubit rotations can be performed at finite singlet-triplet energy splitting J , by combining Z rotations U_Z with rotations U_{XZ} around an axis in the XZ plane. For example, a rotation about the X -axis can be generated by a three-step sequence $U_X = U_{XZ}U_ZU_{XZ}$ [9, 15].

III. TWO-QUBIT GATE

As the qubit is encoded in the (1,1) charge singlet state $|S\rangle$ and triplet state $|T\rangle$, we can switch on the interaction between neighboring qubits by simultaneously changing them to charge state (0,2) only when they are both initially in the singlet state $|S\rangle$. Assume that $\varepsilon = -E_c/2$ and each molecule is initialized in (1,1) charge state. The Coulomb interaction between the two nearest-neighbor qubits can be directly described by the Hamiltonian:

$$H_{int} = \text{diag}\{H_{int_0}, H_{int_0}, H_{int_0}, H_{int_0}\} \quad (3)$$

in the basis $|TT\rangle$, $|T\tilde{S}\rangle$, $|\tilde{S}T\rangle$ and $|\tilde{S}\tilde{S}\rangle$, where

$$H_{int_0} = \frac{1}{4\pi\epsilon} \left(\frac{2e^2}{b} + \frac{2e^2}{\sqrt{a^2 + b^2}} \right). \quad (4)$$

Here ϵ is dielectric constant of GaAs, a is the distance between the dots inside each molecule and b is the distance between neighboring molecules.

When ε is adiabatically swept from $-E_c/2$ towards $E_c/2$, the double-dot state initially in singlet state $|S\rangle$ will evolve as $|\tilde{S}\rangle$ of Eq. 1. The triplet state $|T\rangle$ will remain unchanged in the charge state $(1,1)$. When $\varepsilon = E_c/2$, the state $|S\rangle$ evolves into $(0,2)$ charge state $|S'\rangle$. Then the interaction between these two molecules can be written as:

$$H'_{int} = \text{diag} \{H_{int_0}, H_{int_0}, H_{int_0}, H_{S'S'}\} \quad (5)$$

in the basis $|TT\rangle, |T\tilde{S}\rangle, |\tilde{S}T\rangle, |\tilde{S}\tilde{S}\rangle$, where $H_{S'S'} = e^2/(\pi\epsilon b)$.

Eliminating a constant background interaction H_{int_0} , we get an effective two-molecule interaction:

$$\Delta H_{int} = H'_{int} - H_{int} = \text{diag} \{0, 0, 0, H_{cc}\}, \quad (6)$$

which can be switched on by sweeping ε from $-E_c/2$ towards $E_c/2$. Here $H_{cc} = H_{S'S'} - H_{int_0}$ can be regarded as the differential cross-capacitance energy between the two double-dot systems. It is noted that the effective interaction is switched on whenever the state $|\tilde{S}\rangle$ includes the component of $(0,2)$ charge state[18]. The differential cross-capacitance energy H_{cc} can thus be written as a function of θ :

$$H_{cc} = \frac{|\sin\theta|^2}{4\pi\epsilon} \left(\frac{2e^2}{b} - \frac{2e^2}{\sqrt{a^2 + b^2}} \right). \quad (7)$$

When $\varepsilon = -E_c/2$, $\theta \rightarrow 0$ and H_{cc} is off. When $\varepsilon = E_c/2$, $\theta \rightarrow \pi/2$ and H_{cc} is maximal.

Combined with some single qubit operations, which have been shown available for double-dot molecule in Section.II, we can construct any two-qubit gate and realize universal quantum computation with the present Ising-model effective interaction. For example the controlled-not gate can be achieved with two single-qubit Hadamard operations σ_H and a two-qubit operation $U(t_0) = \text{diag} \{1, 1, 1, -1\}$ as

$$U_{C-NOT} = \{I_1 \otimes \sigma_{H_2}\} U(t_0) \{I_1 \otimes \sigma_{H_2}\}. \quad (8)$$

By choosing a proper interaction time $t = t_0$ for $H_{cc}t/\hbar = \pi, 3\pi, 5\pi, \dots$, we can get the two-qubit operation $U(t_0)$ directly from the present effective interaction:

$$U(t) = \exp \left(\frac{i\Delta H_{int}t}{\hbar} \right). \quad (9)$$

After interaction time t_0 , the ε should be in the value of $-E_c/2$ to completely switch off the effective interaction.

Comparing with the one-dimensional alignment of all quantum dots, the present two-dimensional architecture

can greatly simplify the interaction between the neighboring quantum molecules as Fig.1(b): there is effective interaction only when the two neighboring molecules are both in the charge state $(0,2)$. We can switch on the interaction between any two neighboring qubits (qubit i and $i+1$) by simultaneously changing their charge state from $(1,1)$ to $(0,2)$. Other neighboring qubits such as qubit $i-1$ and $i+2$ are kept in the charge state $(1,1)$ so that they can not be infected by the operations on qubit i and $i+1$. It is noted the Coulomb interaction between two electrons inside each qubit can also be neglected. Only the interaction between the nearest-neighbor molecules is included in the previous protocols. It can be shown that the interaction between non-nearest-neighbor qubits can be neglected safely [15, 16, 18, 19].

If all the quantum dots are arranged in line as Fig.1(a), we can get an effective interaction between neighboring quantum molecules by sweeping the two logical qubits into charge state $(0,2)$ and $(2,0)$ respectively. In this case, the notation (n_L, n_R) indicates n_L electrons on the “left” dot of each qubit and n_R electrons on the “right” dot. In the basis $|TT\rangle, |T\tilde{S}\rangle, |\tilde{S}T\rangle, |\tilde{S}\tilde{S}\rangle$, the interaction between two neighboring qubits i and $i+1$ can be written in the form:

$$H'_{int} = \text{diag} \{H'_{int_0}, H'_{int_0} + E, H'_{int_0} + E, H'_{int_0} + E'\}, \quad (10)$$

where H'_{int_0} is the interaction between two quantum molecules which are both in the $(1,1)$ charge states; E is the Coulomb interaction energy change when qubit i (or $i+1$) is swept from the charge state $(1,1)$ to $(0,2)$ (or $(2,0)$); E' is the energy change when qubit i and $i+1$ are swept from the charge state $(1,1)$ to $(0,2)$ and $(2,0)$ respectively. When there are only two logical qubits, this interaction can be used to get two-qubit gates [15, 16]. However, for the scalable quantum computation, we can not exclusively switch on interaction between two neighboring qubits without influencing other neighboring qubits. For example, when we switch on interaction H'_{int} between qubit i and $i+1$, there is also an effective interaction between qubit $i-1$ and i (or qubit $i+1$ and $i+2$):

$$H''_{int} = \text{diag} \{H'_{int_0}, H'_{int_0}, H'_{int_0} - E, H'_{int_0} - E\}. \quad (11)$$

As we need to change the molecule charge state in the measurement process, the kind of unavoidable effect from other neighboring qubits will be switched on and influence the single qubit readout for this architecture that all quantum dots are arranged in line.

Although the quantum dots are arranged in two-dimensional architecture, we have only considered a one-dimensional logical qubits or quantum molecules chain in the above discussion. Actually, we can scale the logical qubits to a two-dimensional array when bilayer 2-dimensional-electron-gas (2DEG) are used to form quantum molecule[19, 20, 21] as shown in Fig.3. Each molecule comprises one quantum dot in the upper 2DEG

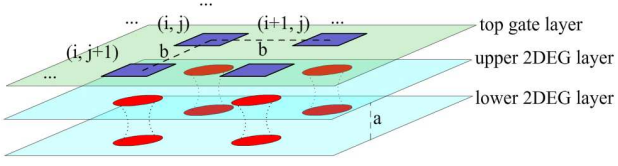


FIG. 3: Schematic diagram of two-dimensional array when bilayer 2-dimensional-electron-gas (2DEG) are used to form quantum molecule. The squares in the top gate layer and circles in the two 2DEG layers stand for controlling gates and quantum dots respectively.

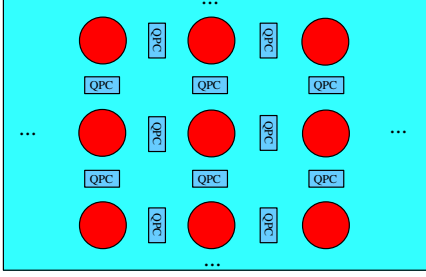


FIG. 4: The structure diagram of single-qubit and two-qubit measurement in the lower 2DEG layer. The quantum point contact (QPC) is used to detect the distribution of two electrons in the nearby double-dot molecule.

and another one in the lower 2DEG. The two quantum dots of upper and lower layers are tunneling coupled to form one quantum molecules. These molecules can also be the self-assembled quantum dot pillars, which also include two quantum dots respectively in the upper and lower part of the pillar[19, 22].

IV. SINGLE QUBIT READOUT AND BELL STATE MEASUREMENT

As the $(1, 1)$ charge state $|S\rangle$ and the $(0, 2)$ charge state $|S'\rangle$ can transform mutually by adjusting the bias voltage ε , the single qubit readout for the present double-dot molecule can be made by a quantum point contact (QPC) placed near one of the quantum dot as shown in Fig.4 [6, 8]. From the current through QPC, we can know the two electrons distribution in the double-dot molecule. If the molecule is in the singlet state $|S\rangle$, it will be changed to the charge state $(0, 2)$ and the current of QPC will be lower, when we sweep ε from $-E_c/2$ to $E_c/2$. If the molecule is in the triplet state $|T\rangle$, it will stay in the charge state $(1, 1)$ and the current of QPC will be unchanged when sweeping ε . As the interaction between neighboring qubits will be switched on when they are both in the charge state $(0, 2)$, we will avoid to perform single qubit measurement on two neighboring molecules simultaneously.

With the present architecture of quantum molecules, we can also perform Bell state measurement for these

qubits encoded in double-dot singlet and triplet states, by directly place a QPC in the middle of two molecules as shown in Fig.4. Any two-qubit state can be denoted by:

$$\Phi_{12} = p_1\Phi^+ + p_2\Phi^- + p_3\Psi^+ + p_4\Psi^-, \quad (12)$$

where $p_1, p_2, p_3, p_4 \in C$, $|p_1|^2 + |p_2|^2 + |p_3|^2 + |p_4|^2 = 1$. $\Phi^\pm = (|TT\rangle \pm |SS\rangle)/\sqrt{2}$ and $\Psi^\pm = (|TS\rangle \pm |ST\rangle)/\sqrt{2}$ are the four Bell states. We can detect the charge state of the quantum dots through the QPC current I . Simultaneously sweep the bias voltage ε of the two qubits, which need to be measured, from $-E_c/2$ to $E_c/2$. Due to the distribution of the four electrons in the two molecules, the QPC current I can thus have three different values: the current I is kept in the value I_{\max} , which means the two qubits are both in the charge state $(1, 1)$; the current I is changed to the value I_{\min} , which means that both qubits are in the charge state $(0, 2)$; the current I gets a value I_{mid} smaller than I_{\max} but larger than I_{\min} , which corresponds to the case that one of the two qubits is in the charge state $(0, 2)$.

As the effective interaction will be switched on when the two qubits are both in the charge state $(0, 2)$, the two-qubit state Φ_{12} will evolve as in the following form when sweeping ε from $-E_c/2$ to $E_c/2$:

$$\begin{aligned} \Phi'_{12} = & \frac{p_1}{\sqrt{2}} (|TT\rangle + e^{i\varphi} |\tilde{S}\tilde{S}\rangle) + \frac{p_2}{\sqrt{2}} (|TT\rangle - e^{i\varphi} |\tilde{S}\tilde{S}\rangle) \\ & + \frac{p_3}{\sqrt{2}} (|T\tilde{S}\rangle + |\tilde{S}T\rangle) + \frac{p_4}{\sqrt{2}} (|T\tilde{S}\rangle - |\tilde{S}T\rangle) \end{aligned} \quad (13)$$

where $\varphi = \int_0^{t_m} H_{cc} dt$, t_m represents the time that ε leaves the value $-E_c/2$. The effective interaction only adds a phase to the component $|\tilde{S}\tilde{S}\rangle$. As it can only be switched on only when the two qubits are both in the state $|SS\rangle$, it has no influence to Ψ^\pm . If the QPC current I gets the value I_{\max} , the two qubits are thus both in the charge state $(1, 1)$. This means that the two qubits are in the state $|TT\rangle$. The current I_{\min} means that the two qubits are both in the charge state $(0, 2)$. Then we can determine that the two-qubit state is in the state $|\tilde{S}\tilde{S}\rangle$, which is evolved from the initial state $|SS\rangle$.

If the QPC current $I = I_{\text{mid}}$, we can know that the one of the two qubits is in the charge state $(0, 2)$. This is the case that the two qubits are in the state $(|T\tilde{S}\rangle + |\tilde{S}T\rangle)/\sqrt{2}$ or $(|T\tilde{S}\rangle - |\tilde{S}T\rangle)/\sqrt{2}$, which respectively evolve from the initial two-qubit state Ψ^+ or Ψ^- . With this step of QPC measurement, we can get the parity information of these two qubits. In the case of $I = I_{\text{mid}}$, we sweep the ε of the both two qubits back to $-E_c/2$ from $E_c/2$ and then perform Hadamard operation on the two qubits in turn. It is ensured that the two qubits can not be simultaneously both in the charge state $(0, 2)$ in the operations. The two Hadamard operations will rotate Ψ^\pm respectively into the state Φ^- and $-\Psi^-$ [23]. Sweep the ε of the both two qubits from

$-E_c/2$ to $E_c/2$ and make QPC measurement again. As the above measurement, the QPC current I' can also have three different values: I_{\max} , I_{mid} , I_{\min} . From the values of I and I' , we can determine two Bell state Ψ^+ and Ψ^- as shown in Table.I. We can sweep the molecule charge state back to (1,1) after this measurement. It is noted that this Bell state measurement is not a completed one, and we can only distinguish two of the four Bell states. In addition, this measurement can also be regarded as a processing of generating Bell state Ψ^- , as the two quantum molecules which are measured in this Bell state can be used in future applications.

| | I | I' |
|----------|--------------------------|--------------------------|
| Φ^+ | I_{\max} OR I_{\min} | — |
| Φ^- | I_{\max} OR I_{\min} | — |
| Ψ^+ | I_{mid} | I_{\max} OR I_{\min} |
| Ψ^- | I_{mid} | I_{mid} |

TABLE I: The states of the current through the QPC corresponding to each Bell state.

V. DISCUSSION AND CONCLUSION

By encoding in singlet and triplet states, qubits are protected from low-frequency noise and the effect of homogeneous hyperfine interactions for double dots. Recent experiments have demonstrated that the coherence time of the singlet and triplet states can be about 10ns , which can even be increased to $1\mu\text{s}$ with spin-echo techniques[6, 8]. As the rapid adiabatic passage of ε is required to be fast relative to the nuclear mixing time $\sim \hbar/(g^*\mu_B B_{\text{nuc}})$ but slow with respect to the tunnel coupling $T_c \sim 0.01\text{meV}$, the ε sweeping speed is about 5meV/ns in these experiments. If the quantum dot of molecule has a diameter of 100nm , the charge energy $E_c \sim 5\text{meV}$ and sweeping ε from $-E_c/2$ to $E_c/2$ needs a time of about 1ns . For quantum molecules based on bilayer 2DEG as Fig.3, the distance between the dou-

ble dots of each molecules $a = 20\text{nm}$ and the distance between neighboring molecules $b = 10a = 200\text{nm}$ (in order to safely neglect the effect from the interaction between non-nearest-neighbor qubits), and we need a time of about 1ns to achieve a two-qubit controlled phase operation $U = \text{diag}\{1, 1, 1, -1\}$ [18]. Actually, in the previous protocols of arranging four quantum dots of two molecules in line, qubits are similarly coupled by Coulomb interaction[15, 16]. The two-qubit operations may also need a time of about 1ns . Therefore, we still need to increase the coherence time or increase the interaction strength, even qubit is encoded in singlet and triplet states for these quantum computation schemes exploring Coulomb interaction to realizing two-qubit gates. Since the QPC measurement needs a time of about $1\mu\text{s}$, QPC measurement can be implemented only once within the coherence time. Thus only partial Bell state measurement for qubit encoded in singlet and triplet state may be realizable with the present experiment conditions. Generally, we can also distinguish the four Bell states by firstly transfer them into four product states respectively and then simultaneously performing QPC measurement on each qubit within the coherence time.

In conclusion, we have proposed a quantum computation architecture based on double-dot quantum molecules. As the qubit is encoded in the (1,1) charge singlet state $|S\rangle$ and triplet state $|T\rangle$, we can simplify the Coulomb interaction to a switchable Ising interaction in the present architecture. Compared with the previous schemes, the effective Ising interaction can be switched on and off between any two neighboring qubits without affecting other neighboring qubits. A Bell-state measurement scheme is also presented for qubit encoded in the singlet and triplet state. Universal quantum gates can be performed by only tuning the potential offset between the two dots of each molecule, where the time-dependent control of the tunnel coupling between the dots is eliminated.

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